# HOSSAM GHANEM

## (14) 7.1 The Derivative Of The Inverse Function (B)

**THEOREM** LET A function $f$ is differentiable and has a inverse $f^{-1}$ THEN

| | |
|---|---|
| $[f^{-1}(c)]^\backslash = \dfrac{1}{f^\backslash\left(f^{-1}(c)\right)}$ | $[f^{-1}(x)]^\backslash = \dfrac{1}{f^\backslash\left(f^{-1}(x)\right)}$ |

**Example 1**

Let $f(x) = \tan^{-1} x + \tan^{-1}(\ln x) - \dfrac{\pi}{4}$ , $x > 0$. 28 July 2006 A

(a) Show that $f$ is one-to one over the interval $(0, \infty)$ .
(b) Find the domain of $f^{-1}$ .
(c) Show that $P(0, 1)$ is on the graph of $f^{-1}$ and find the slope of the tangent line at $P$.

## Solution

$Df = (0, \infty)$

$$f^\backslash(x) = \frac{1}{1+x^2} + \frac{1}{1+(\ln x)^2}\cdot\frac{1}{x} > 0 \qquad \forall\, x \in (0, \infty)$$

$\therefore f \uparrow$ $\therefore f\ 1-1$ $\therefore f$ has an inverse

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \tan^{-1} x + \tan^{-1}(\ln x) - \frac{\pi}{4} = 0 - \frac{\pi}{2} - \frac{\pi}{4} = \frac{-3\pi}{4}$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \tan^{-1} x + \tan^{-1}(\ln x) - \frac{\pi}{4} = \frac{\pi}{2} + \frac{\pi}{2} - \frac{\pi}{4} = \frac{3\pi}{4}$$

$\therefore D_f = (0, \infty)$ $\qquad R_f = \left(\dfrac{-3\pi}{4}, \dfrac{3\pi}{4}\right)$

$\therefore D_{f^{-1}} = \left(\dfrac{-3\pi}{4}, \dfrac{3\pi}{4}\right)$ $\qquad R_{f^{-1}} = (0, \infty)$

$$f(1) = \tan^{-1} 1 + \tan^{-1}(\ln 1) = \frac{\pi}{4} + 0 - \frac{\pi}{4} = 0$$

$\therefore (1,0)$ on the graph of $f$

$\therefore (0,1)$ on the graph of $f^{-1}$

$$f^\backslash(1) = \frac{1}{1+1} + 1 = \frac{3}{2}$$

$$m = [f^{-1}(0)]^\backslash = \frac{1}{f^\backslash\left(f^{-1}(0)\right)} = \frac{1}{f^\backslash(1)} = \frac{2}{3}$$

## Example 2

40 August 7 , 2011

$(1 + 1 + 2$ pts $)$ Let $f(x) = \tan^{-1}(x + \ln x)$ , $x > 0$ .

(a) Show that $f^{-1}$exists.

(b) Find the domain of $f^{-1}$

(c) Show that the point $\left(\frac{\pi}{4}, 1\right)$ is on the graph of $f^{-1}$ , and find the equation of the tangent line to the graph of $f^{-1}$ at $\left(\frac{\pi}{4}, 1\right)$

# Solution

$Df = (0, \infty)$

$f(x) = \tan^{-1}(x + \ln x)$

$$f^{\backslash}(x) = \frac{1 + \frac{1}{x}}{1 + (x + \ln x)^2} > 0 \quad \forall\ x \in (0, \infty)$$

$\therefore f \uparrow \quad \therefore f\ 1 - 1 \quad \therefore f$ has an inverse $\quad \therefore f^{-1}$exists

$$f(1) = \tan^{-1}(1 + \ln 1) = \tan^{-1}(1) = \frac{\pi}{4}$$

$\therefore \left(1, \frac{\pi}{4}\right)$ on the graph of $f$

$\therefore \left(\frac{\pi}{4}, 1\right)$ on the graph of $f^{-1}$

$$f^{-1}\left(\frac{\pi}{4}\right) = 1$$

$$f^{\backslash}(1) = \frac{1 + \frac{1}{1}}{1 + (1 + \ln 1)^2} = \frac{2}{2} = 1$$

$$m = \left[f^{-1}\left(\frac{\pi}{4}\right)\right]^{\backslash} = \frac{1}{f^{\backslash}\left(f^{-1}\left(\frac{\pi}{4}\right)\right)} = \frac{1}{f^{\backslash}(1)} = \frac{1}{1} = 1$$

$m = 1 \qquad P\left(\frac{\pi}{4}, 1\right)$

$y - y_1 = m(x - x_1)$

$y - 1 = x - \frac{\pi}{4}$

## Example 3

Given 15 May 1999 A

$f(x) = xe^{x^2} + 2\sin^{-1} x + \sinh x^3 + 1 , \quad -1 \le x \le 1.$
Show that $f^{-1}$ exists and find
the equation of the tangent line to the graph of $f^{-1}$ at the point $P(1,0)$

## Solution

$Df = [-1,1]$

$f(x) = xe^{x^2} + 2\sin^{-1} x + \sinh x^3 + 1$

$f^{\backslash}(x) = e^{x^2} + 2x^2e^{x^2} + \dfrac{2}{\sqrt{1-x^2}} + 3x^2\cosh x^3 > 0 \quad \forall\, x \in (-1,1)$

$\therefore f \uparrow \qquad \therefore f\ 1-1 \qquad \therefore f$ has an inverse $\qquad \therefore f^{-1}$ exists

$f^{\backslash}(0) = 1 + 0 + \dfrac{2}{\sqrt{1}} + 0 = 3$

the point $P(1,0)$ $\therefore f^{-1}(1) = 0$

$m = [f^{-1}(1)]^{\backslash} = \dfrac{1}{f^{\backslash}(f^{-1}(1))} = \dfrac{1}{f^{\backslash}(0)} = \dfrac{1}{3}$

$\therefore m = 3 \qquad P(1,0)$

$y - y_1 = m(x - x_1)$

$y = 3(x - x_1)$

## Example 4

Let $f(x) = \displaystyle\int_0^x \tan(t)\sec^2(t)\ dt \quad , \quad 0 \le x \le \pi/2$ 27 June 2006 A

Find (a) $f'(x)$ (b) $f(\pi/4)$ (c) $(f^{-1})^{\backslash}\left(\dfrac{1}{2}\right)$

## Solution

$f^{\backslash}(x) = \tan(x)\sec^2(x)$

$f\left(\dfrac{\pi}{4}\right) = \displaystyle\int_0^{\frac{\pi}{4}} \tan(x)\sec^2(x)\ dx$

Let $u = \tan x \qquad \therefore du = \sec^2(x)\, dx$

at $x = 0 \ \to\to u = 0 \qquad$ at $x = \dfrac{\pi}{4} \ \to\to u = 1$

$f\left(\dfrac{\pi}{4}\right) = \displaystyle\int_0^1 u\ du = \dfrac{1}{2}\Big[u\Big]_0^1 = \dfrac{1}{2}(1-0) = \dfrac{1}{2}$

$f\left(\dfrac{\pi}{4}\right) = \dfrac{1}{2} \qquad \therefore f^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{4}$

$f^{\backslash}\left(\dfrac{\pi}{4}\right) = \tan\left(\dfrac{\pi}{4}\right)\sec^2\left(\dfrac{\pi}{4}\right) = 1 \cdot 2 = 2$

$(f^{-1})^{\backslash}\left(\dfrac{1}{2}\right) = \dfrac{1}{f^{\backslash}\left(f^{-1}\left(\frac{1}{2}\right)\right)} = \dfrac{1}{f^{\backslash}\left(\frac{\pi}{4}\right)} = \dfrac{1}{2}$

## Example 5

32 January 2009 A

Let $f$ be a one-to-one differentiable function such that $f(2) = 3$ , $f'(2) = 5$ , $f'(3) = 4$. If $g = f^{-1}$ , what is $g'(3)$ ?

Solution

$f(2) = 3$

$f^{-1}(3) = 2 \qquad , g(3) = 2$

$$g^{\backslash}(3) = [f^{-1}(3)]^{\backslash} = \frac{1}{f^{\backslash}[f^{-1}(3)]} = \frac{1}{f^{\backslash}(2)} = \frac{1}{5}$$

## Example 6

28 April 2009 A

Let $g$ be the inverse function of $f$ and $h(x) = xg^2(x)$.
If $f(1) = 3$ and $f'(1) = 2$ then find $h'(3)$.

Solution

$f(1) = 3 \quad \rightarrow\rightarrow\rightarrow \quad f^{-1}(3) = 1 \quad \rightarrow\rightarrow\rightarrow \quad g(3) = 1$

$$g^{\backslash}(3) = [f^{-1}(3)]^{\backslash} = \frac{1}{f^{\backslash}[f^{-1}(3)]} = \frac{1}{f^{\backslash}(1)} = \frac{1}{2}$$

$h(x) = xg^2(x)$

$h^{\backslash}(x) = g^2(x) + 2x\, g(x)\, g^{\backslash}(x)$

$$h^{\backslash}(3) = g^2(3) + 2(3)\, g(3)\, g^{\backslash}(3) = 1 + 6 \cdot 1 \cdot \frac{1}{2} = 4$$

# Homework

| | | |
|---|---|---|
| 1 | Given that $f(x) = 2x^3 + 5x + 3$,<br>find the slope of the tangent line to the graph of $f^{-1}(x)$ when $x = 10$ | 12 July 2000 A |
| 2 | Given the function $f(x) = x^5 + x^3 + 2x - 2$.<br>(a) Show that $f$ is one-to-one.<br>(b) Show that the point $P(2, 1)$ is on the graph of $f^{-1}$, and find the slope of the tangent line to the graph of $f^{-1}$ at $P$. | 23 January 2005 A |
| 3 | Find the slope of the tangent line to the graph of $f^{-1}$ at the point $(3, 1)$, where $f(x) = x^7 + 3x^5 + x - 2$ | 11 October 1999 |
| 4 | Let $f(x) = 3x^3 - 2x^{-1} - 1$ , $x > 0$<br>(a) Show that $f$ is one-to-one on $(0, \infty)$<br>(b) Determine the domain of $f^{-1}$<br>(c) Find $(f^{-1})'(0)$ | 34 August 2009 A |
| 5 | Let $f(x) = 2^{3x} + 2 \cdot 5^x - 5$<br>Find an equation of the tangent line to the graph of $f^{-1}$ at the point $P(-2, 0)$ | 7 July 1997 |
| 6 | Let $f(x) = 5e^x - 2e^{-x} - 2$, $(x \in (-\infty, \infty))$.<br>Find the slope of the tangent line to the graph of $f^{-1}$ at the point $(1, 0)$ | 3 Nov. 1994 |
| 7 | Let $h(x) = 2x^3 + 3^{2x}$.<br>Find an equation for the tangent to the graph of $y = h^{-1}(x)$ at the point (1,0). | 9 October 1998 |

# Homework

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|---|---|
| 8 | 2 March 1993<br>Show that the function $f(x) = (\tan^{-1}x - x)$ , $(x \in R)$ is decreasing<br>Find an equation of the tangent line to the graph of $f^{-1}$ at the point $P\left(\frac{\pi-4}{4}, 0\right)$ |
| 9 | Consider $f(x) = \tan^{-1}x + \cosh x$ , $x \geq 0.$ 19 March 2006 A<br>a. Prove that $f$ is one-to-one.<br>b. Find the slope of the tangent line to the curve $y = f^{-1}(x)$ at the point ( 1 , 0 ). |
| 10 | Let $f(x) = e^x - \tan^{-1}x$ , $x > 0.$ 16 November 2004<br>(a) Prove that $f$ is $1-1$.<br>(b) Find the domain and range of $f^{-1}$<br>(c) Find an equation for the tangent line to the curve $y = f^{-1}(x)$ at the point $(e - \pi/4\ ,\ 1)$ |
| 11 | Let $f(x) = \tan^{-1}(\sqrt{\ln x}) + \frac{\pi}{4}$ 16 December 1999 A<br>(a) What is the domain of $f$ ? Show that $f$ is one-to-one on its domain<br>(b)Show that $P\left(\frac{\pi}{2}, e\right)$ is on the graph of $f^{-1}$ and find the equation of the tangent line to the graph of $f^{-1}$ at $P$. (2 points) |
| 12 | Let $f(x) = \tan^{-1}(x + 2\ln x)$, where $x > 0.$ 26 January 2006 A<br>(a) Show that $f$ is one-to-one on its domain .<br>(b) Find the range of $f$ .<br>(c) Find the equation of the tangent line to the graph of $f^{-1}$ at the point $\left(\frac{\pi}{4}, 1\right)$ |

# Homework

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|---|---|
| 13 | Let $f(x)=\ln x+\dfrac{1}{\ln x}$ — 26 July 2008 A<br>i. Find the domain of $f$ .<br>ii. Show that $f$ is one-to-one in the interval $(e,\infty)$ .<br>iii. Find the slope of the tangent line to the graph of $f^{-1}(x)$ at the point $P\left(\frac{5}{2},e^{2}\right)$ |
| 14 | Let $f(x)=\cos^{-1}(e^{x})-\ln(x+2)-\cos^{-1}(e^{-1})$. — 24 May 2005 A<br>(a) Find the domain of $f$<br>(b) Show that $f$ has an inverse.<br>(c) Find the slope of the tangent line to the graph of $f^{-1}$ at the point $P(0,-1)$ |
| 15 | Consider $f(x)=(1-x)^{\ln(2x+1)}$ — 34 August 2009 A<br>(a) Find the domain of $f$<br>(b) Find $f^{\backslash}(1/2)$ |
| 16 | Let $f(x)=e^{x}+e^{\tan^{-1}x}$ , $-\infty<x<\infty$. — 24 March 2008 A<br>i) Show that $f^{-1}$ exits and find its domain.<br>ii) Show that $P(2,0)$ is on the graph of $f^{-1}$ and find the slope of the tangent line to the graph of $f^{-1}$ at $P(2,0)$. |
| 17 | Let $f(x)=\ln(\cos x^{3})-2x+1$ where $0\le x\le 1$ — 8 October 1997, 15 July 2003 A<br>a. Show that $f^{-1}$exists<br>b. Find the equation of the tangent line to the graph of $f^{-1}$at point $p(1,\ 0)$ |
| 18 | Let $f(x)=\int_{2}^{x}\frac{1}{t}e^{-t}dt$ , $x>0$ — 30 January 2008<br>(a) Show that $f$ is one-to-one on its domain.<br>(b) Explain why the points $P(0,2)$ is on the graph of $f^{-1}$<br>(c) Find the slope of the tangent line to the graph of $f^{-1}$ at $P(0,2)$. |

# Homework

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|---|---|
| 19 | Let $f(x) = e^{-x} - x$ for $x \in \mathbb{R}$. 31 10 July 2010<br>(a) Show that $f$ is one-to-one on $\mathbb{R}$. [1 mark]<br>(b) Determine the domain of $f^{-1}$ [1 mark]<br>(c) Explain why the point ( 1 , 0 ) is on the graph of $f^{-1}$, and find the slope of the tangent line at this point. [2 mark] |
| 19 | (2+2+2 pts) Let $f(x) = \sin^{-1}(2 - x) + \ln(4 - x^2)$ 33 April 10, 2011<br>(a) Find the domain of $f$ .<br>(b) Show that $f$ has an inverse function.<br>(c) Find the domain of $f^{-1}$ |
| 20 | (1+2+1 pts. ) Let $f(x) = \sqrt{(\ln x)^2 + 3}$ . 36 June 6, 2010<br>(a) Show that $f$ is one-to-one on $[1, \infty)$<br>(b) Find $(f^{-1})'(2)$.<br>(c) Show that $f$ is not one-to-one on $(0, \infty)$ |
| 21 | 38 Jan. 22, 2011<br>(2+1 pts. ) Let $y$ be a function of $x$ defined implicitly by<br>$y^3 + x^2 e^y + x^4 = 1 \quad , \quad x \geq 0$<br>(a) Show that this function is one-to-one .<br>(b) Show that the graph of the inverse function has a vertical tangent at ( 0 , 1 ) |
| 22 | 11 October 1999<br>Show that the function<br>$f(x) = \sqrt{3 + e^{x-1}}$ , $x \in [1, \infty)$ is one-to-one.<br>Find $f^{-1}(x)$ and state its domain and range. |
| 23 | Consider $f(x) = \|x\| \tan^{-1} x$ for $-\infty < x < \infty$ 29 July 2009 A<br>(a) Show that $f$ is one-to-one on $(-\infty, \infty)$<br>(b) Determine the domain of $f^{-1}$<br>(c) Find $(f^{-1})\backslash(\pi/4)$ |

# Homework

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|---|---|
| 24 | Given that $f(x) = x + \ln(e + \tanh x)$. 21 January 2004 A<br><br>(a) Explain why the domain of $f$ is $(-\infty, \infty)$.<br>(b) Show that $f$ is one-to-one on its domain.<br>(c) Show that the point $P(1, 0)$ is on the graph of $f^{-1}$<br>and find the equation of the tangent line to the graph of $f^{-1}$ at $P$. |
| 25 | Given $f(x) = 3x^2 - \sin^{-1}(2^x) + \dfrac{\pi}{6}$ 22 June 2004 A<br><br>(a) Find the domain of $f$<br>(b) Show that $f^{-1}$ exists, and find its domain.<br>(c) Why is the point $P(3, -1)$ on the graph of $f^{-1}$ ?<br>Find the slope of the tangent line to the graph of $f^{-1}$ at $P$. |
| 26 | Let $g(x) = \displaystyle\int_2^x \frac{t\ dt}{e^{t-2} + t^4}$ , $(x \geq 0)$ 6 March 1997<br><br>Show that the $g$ is one-to- one and find the equation of the tangent line to the graph of its inverse function $g^{-1}$ at the point $P(0, 2)$. |
| 27 | Show that if $f$ is one-to-one,<br>then $g(x) = e^{f(x)}$ is also one-to-one 32 January 2009 A |

**23**

Consider $f(x) = |x| \tan^{-1} x$ for $-\infty < x < \infty$ 29 July 2009 A

(a) Show that $f$ is one-to-one on $(-\infty, \infty)$

(b) Determine the domain of $f^{-1}$

(c) Find $(f^{-1})^{\backslash}(\pi/4)$

# Solution

$Df = (-\infty, \infty)$

if $x > 0$ $\quad f(x) = x \tan^{-1} x$

$f^{\backslash}(x) = \tan^{-1} x + \frac{x}{1+x^2} > 0 \qquad \forall\, x \in (0, \infty)$

$\therefore f \uparrow$ on $(0, \infty)$

if $x < 0$ $\quad f(x) = -x \tan^{-1} x$

$f^{\backslash}(x) = -\tan^{-1} x - \dfrac{x}{1+x^2} > 0 \qquad \forall\, x \in (-\infty, 0)$

$\therefore f \uparrow$ on $(-\infty, 0)$

if $x < 0$ $\quad f(x) < 0$

if $x > 0$ $\quad f(x) > 0$

if $a \in (0, \infty)$ and $b \in (-\infty, 0)$ then $f(a) \neq f(b)$

$\therefore f \uparrow$ 0n $(-\infty, \infty)$ $\qquad \therefore f\ 1-1$ $\qquad \therefore f$ has an inverse

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} (-x \tan^{-1} x) = \infty\left(-\frac{\pi}{2}\right) = -\infty$$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} (x \tan^{-1} x) = \infty$$

$\therefore D_f = (-\infty, \infty)$ $\qquad R_f = (-\infty, \infty)$

$\therefore D_{f^{-1}} = (-\infty, \infty)$ $\qquad R_{f^{-1}} = (-\infty, \infty)$

$f(x) = \dfrac{\pi}{4} \quad \to\to \therefore\ |x| \tan^{-1} x = \dfrac{\pi}{4} \quad \to\to x = 1$

$\therefore f(1) = \dfrac{\pi}{4} \qquad \therefore f^{-1}\left(\dfrac{\pi}{4}\right) = 1$

$$f^{\backslash}(1) = \frac{\pi}{4} + \frac{1}{2} = \frac{\pi + 2}{4}$$

$$\left[f^{-1}\left(\frac{\pi}{2}\right)\right]^{\backslash} = \frac{1}{f^{\backslash}\left(f^{-1}\left(\frac{\pi}{2}\right)\right)} = \frac{1}{f^{\backslash}(1)} = \frac{4}{\pi + 2}$$

24

Given that $f(x) = x + \ln(e + \tanh x)$. 21 January 2004 A

(a) Explain why the domain of $f$ is $(-\infty, \infty)$.
(b) Show that $f$ is one-to-one on its domain.
(c) Show that the point $P(1, 0)$ is on the graph of $f^{-1}$
and find the equation of the tangent line to the graph of $f^{-1}$ at $P$.

# Solution

$D_f = (-\infty, \infty)$

$-1 \le \tanh x \le 1$

$e - 1 \le e + \tanh x \le e + 1$

$\therefore e + \tanh x > 0 \qquad \forall\, x \in (-\infty, \infty)$

$\therefore D_f = (-\infty, \infty)$

$$f^{\backslash}(x) = 1 + \frac{\operatorname{sech}^2 x}{e + \tanh x} > 0$$

$\therefore f \uparrow \qquad \therefore f\ 1-1 \qquad \therefore f$ has an inverse

$f(0) = 0 + \ln(e) = 1$

$\therefore$ (0,1) on the graph of $f$

$\therefore$ (1,0) on the graph of $f^{-1}$

$$f^{\backslash}(0) = 1 + \frac{1}{e} = \frac{e+1}{e}$$

$$m = [f^{-1}(1)]^{\backslash} = \frac{1}{f^{\backslash}\left(f^{-1}(1)\right)} = \frac{1}{f^{\backslash}(0)} = \frac{e}{e+1}$$

$$\therefore m = \frac{e}{e+1} \qquad , \qquad P(1, 0)$$

$y - y_1 = m(x - x_1)$

$$y = \frac{e}{e+1}(x - 1)$$

**25**

Given $f(x) = 3x^2 - \sin^{-1}(2^x) + \frac{\pi}{6}$ 22 June 2004 A

(a) Find the domain of $f$
(b) Show that $f^{-1}$ exists, and find its domain.
(c) Why is the point $P(3, -1)$ on the graph of $f^{-1}$ ?
Find the slope of the tangent line to the graph of $f^{-1}$ at $P$.

# Solution

$D_f$

$2^x \leq 1 \quad \rightarrow\rightarrow \quad x \ln 2 \leq 0 \quad \rightarrow\rightarrow \quad x \leq 0$

$\therefore D_f = (-\infty, 0)$

$f^\backslash(x) = 6x - \frac{2^x \ln 2}{\sqrt{1 - 2^{2x}}} < 0 \quad \forall \ x \in (-\infty, 0)$

$\therefore f \downarrow \quad \therefore f\ 1-1 \quad \therefore f$ has an inverse $\quad \therefore f^{-1}$ exists

$R_f$

$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \left(3x^2 - \sin^{-1}(2^x) + \frac{\pi}{6}\right) = \infty$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(3x^2 - \sin^{-1}(2^x) + \frac{\pi}{6}\right) = 0 - \frac{\pi}{2} + \frac{\pi}{6} = \frac{-3\pi}{6} + \frac{\pi}{6} = \frac{-2\pi}{6} = \frac{-\pi}{3}$

$\therefore D_f = (-\infty, 0)$ $\qquad R_f = \left(-\frac{\pi}{3}, \infty\right)$

$\therefore D_{f^{-1}} = \left(-\frac{\pi}{3}, \infty\right)$ $\qquad R_{f^{-1}} = (-\infty, 0)$

$f(-1) = 3(1) - \sin^{-1}\frac{1}{2} + \frac{\pi}{6} = 3 - \frac{\pi}{6} + \frac{\pi}{6} = 3$

$\therefore (-1,3)$ on the graph of $f$

$\therefore (3,-1)$ on the graph of $f^{-1}$

$f^\backslash(-1) = -6 - \frac{\frac{1}{2}\ln 2}{\sqrt{1 - \frac{1}{4}}} = 6 - \frac{\frac{1}{2}\ln 2}{\frac{\sqrt{3}}{2}} = 6 - \frac{\ln 2}{\sqrt{3}} = \frac{6\sqrt{3} - \ln 2}{\sqrt{3}}$

$m = [f^{-1}(3)]^\backslash = \frac{1}{f^\backslash(f^{-1}(3))} = \frac{1}{f^\backslash(-1)} = \frac{\sqrt{3}}{6\sqrt{3} - \ln 2}$

**26**

Let $g(x) = \int_2^x \frac{t\ dt}{e^{t-2} + t^4}$ , $(x \geq 0)$ 6 March 1997

Show that the $g$ is one-to- one and find the equation of the tangent line to the graph of its inverse function $g^{-1}$ at the point $P(0, 2)$.

## Solution

$D_g = [0, \infty)$

$g^{\backslash}(x) = \frac{x}{e^{x-2} + x^4} > 0 \qquad \forall\ x \in [0, \infty)$

$\therefore g \uparrow \qquad \therefore g\ 1-1 \qquad \therefore g$ has an inverse

$\therefore g^{-1}(0) = 2$

$g^{\backslash}(2) = \frac{2}{1+16} = \frac{2}{17}$

$m = [g^{-1}(0)]^{\backslash} = \frac{1}{g^{\backslash}[g^{-1}(0)]} = \frac{1}{g^{\backslash}(2)} = \frac{17}{2}$

$m = \frac{17}{2} \qquad P(0, 2)$

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{17}{2} x$

**27**

Show that if $f$ is one-to-one, then $g(x) = e^{f(x)}$ is also one-to-one. 32 January 2009 A

Solution

$\because f\ 1-1$

$\therefore$ if $x_1 \neq x_2$

$\therefore f(x_1) \neq f(x_2)$

$\therefore e^{f(x_1)} \neq e^{f(x_2)}$

$\therefore g(x_1) \neq g(x_2)$

$g(x) = e^{f(x)} \quad 1-1$